

## MATHEMATICS

## READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50 .
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

1 There are 18 people in Millie's class. To choose a person at random she numbers the people in the class from 1 to 18 and presses the random number button on her calculator to obtain a 3-digit decimal. Millie then multiplies the first digit in this decimal by two and chooses the person corresponding to this new number. Decimals in which the first digit is zero are ignored.
(i) Give a reason why this is not a satisfactory method of choosing a person.

Millie obtained a random sample of 5 people of her own age by a satisfactory sampling method and found that their heights in metres were $1.66,1.68,1.54,1.65$ and 1.57 . Heights are known to be normally distributed with variance $0.0052 \mathrm{~m}^{2}$.
(ii) Find a $98 \%$ confidence interval for the mean height of people of Millie's age.

2 A computer user finds that unwanted emails arrive randomly at a uniform average rate of 1.27 per hour.
(i) Find the probability that more than 1 unwanted email arrives in a period of 5 hours.
(ii) Find the probability that more than 850 unwanted emails arrive in a period of 700 hours.

3 An airline knows that some people who have bought tickets may not arrive for the flight. The airline therefore sells more tickets than the number of seats that are available. For one flight there are 210 seats available and 213 people have bought tickets. The probability of any person who has bought a ticket not arriving for the flight is $\frac{1}{50}$.
(i) By considering the number of people who do not arrive for the flight, use a suitable approximation to calculate the probability that more people will arrive than there are seats available.

Independently, on another flight for which 135 people have bought tickets, the probability of any person not arriving is $\frac{1}{75}$.
(ii) Calculate the probability that, for both these flights, the total number of people who do not arrive is 5 .

4 It is not known whether a certain coin is fair or biased. In order to perform a hypothesis test, Raj tosses the coin 10 times and counts the number of heads obtained. The probability of obtaining a head on any throw is denoted by $p$.
(i) The null hypothesis is $p=0.5$. Find the acceptance region for the test, given that the probability of a Type I error is to be at most 0.1 .
(ii) Calculate the probability of a Type II error in this test if the actual value of $p$ is 0.7 .

5 The masses of packets of cornflakes are normally distributed with standard deviation 11 g . A random sample of 20 packets was weighed and found to have a mean mass of 746 g .
(i) Test at the $4 \%$ significance level whether there is enough evidence to conclude that the population mean mass is less than 750 g .
(ii) Given that the population mean mass actually is 750 g , find the smallest possible sample size, $n$, for which it is at least $97 \%$ certain that the mean mass of the sample exceeds 745 g .

6 The continuous random variable $X$ has probability density function given by

$$
\mathrm{f}(x)= \begin{cases}\frac{1}{3} x(k-x) & 1 \leqslant x \leqslant 2 \\ 0 & \text { otherwise }\end{cases}
$$

(i) Show that the value of $k$ is $\frac{32}{9}$.
(ii) Find $\mathrm{E}(X)$.
(iii) Is the median less than or greater than 1.5? Justify your answer numerically.

7 (a) Random variables $Y$ and $X$ are related by $Y=a+b X$, where $a$ and $b$ are constants and $b>0$. The standard deviation of $Y$ is twice the standard deviation of $X$. The mean of $Y$ is 7.92 and is 0.8 more than the mean of $X$. Find the values of $a$ and $b$.
(b) Random variables $R$ and $S$ are such that $R \sim \mathrm{~N}\left(\mu, 2^{2}\right)$ and $S \sim \mathrm{~N}\left(2 \mu, 3^{2}\right)$. It is given that $\mathrm{P}(R+S>1)=0.9$.
(i) Find $\mu$.
(ii) Hence find $\mathrm{P}(S>R)$.

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